Singapore Examinations and Assessment Board

Cambridge Assessment International Education

Singapore–Cambridge General Certificate of Education Advanced Level Higher 3 (2025)

Mathematics (Syllabus 9820)

(First year of examination in 2025)

CONTENTS

PREAMBLE

Mathematicians work with precise definitions, make conjectures, prove new results and solve problems. They are concerned with the properties of mathematical objects and the applications of abstract ideas and models to solve problems. Mathematical truths and solutions come from rigorously constructed arguments called proofs and mathematically sound procedures and steps. The work of mathematicians has impact in different fields, far beyond just sciences and engineering.

H3 Mathematics provides students, who intend to pursue mathematics at the university, with an insight into the practice of a mathematician. It equips students with the knowledge and skills to understand and write mathematical statements, proofs and solutions, and the techniques and results that come in helpful in their work. Students will develop these competencies through proving mathematical results and solving non-routine mathematical problems in the course of the learning.

SYLLABUS AIMS

The aims of H3 Mathematics are to enable students to:

(a) acquire advanced problem-solving skills and methods of proof by learning useful mathematical results and techniques to solve non-routine problems and prove statements

(b) develop rigour in mathematical argument and precision in the use of mathematical language through the writing and evaluation of mathematical proofs and solutions

(c) experience and appreciate the practice, value and rigour of mathematics as a discipline.

ASSESSMENT OBJECTIVES (AO)

The assessment will test candidates' abilities to:

AO1 **Use mathematical techniques and procedures**
- recall facts, formulae and notation and use them directly
- Read and use information from tables, graphs, diagrams and texts
- carry out straightforward mathematical procedures

AO2 **Formulate and solve problems including those in real-world contexts**
- select relevant mathematical concept or strategy to apply
- formulate problems into mathematical expressions or models
- integrate mathematical concepts to solve mathematical problems
- translate between equivalent forms of mathematical expressions or statements
- interpret results in the context of a given problem

AO3 **Reason and communicate mathematically**
- explain the choice of mathematical models or strategies
- make deductions, inferences and generalisations
- formulate conjectures and justify mathematical statements
- construct mathematical arguments and proofs

Approximate weightings for the assessment objectives are as follows:

AO1	10 marks
AO2	35 marks
AO3	35 marks

USE OF A GRAPHING CALCULATOR (GC)

The use of an approved GC without computer algebra system will be expected. The examination papers will be set with the assumption that candidates will have access to GC. As a general rule, unsupported answers obtained from GC are allowed unless the question states otherwise. Where unsupported answers from GC are not allowed, candidates must present the mathematical steps using mathematical notations and not calculator commands. For questions where graphs are used to find a solution, candidates should sketch these graphs as part of their answers. Incorrect answers without working will receive no marks. However, if there is written evidence of using GC correctly, method marks may be awarded.

Candidates should be aware that there are limitations inherent in GC. For example, answers obtained by tracing along a graph to find roots of an equation may not produce the required accuracy.

LIST OF FORMULAE AND RESULTS

Candidates will be provided in the examination with a list of formulae and results.

INTEGRATION AND APPLICATION

Notwithstanding the presentation of the topics in the syllabus document, it is envisaged that some examination questions may integrate ideas from more than one topic, and that topics may be tested in the contexts of problem solving and application of mathematics.

SCHEME OF EXAMINATION PAPERS

For the examination in H3 Mathematics, there will be one 3-hour paper marked out of 80. The paper will consist of 6 questions of different lengths. Questions 1 to 5 will be worth 10 to 14 marks each. Question 6 will be worth 16 to 20 marks and will require candidates to read and respond to a short mathematical text.

Candidates will be expected to answer **all** questions.

CONTENT OUTLINE

Knowledge of the content of H2 Mathematics is assumed.

	Topics/Sub-topics	Content

1	Mathematical Statements	Include:
		• Definition, Proposition and Theorem
		• Conditionals
		  – if … then …
		  – … if and only if …
		  – Necessary
		  – Sufficient
		• Quantifiers such as
		  – There exists (a unique) …
		  – For all …
		• Logical connectives
		  – ‘and’, ‘or’, ‘not’, ‘implies’
		• Converse
		• Inverse
		• Contrapositive
		• Negation

2	Mathematical Proofs and Reasoning Principles	Include: • Direct proof • Disproof by counterexample • Proof by contradiction • Proof of existence • Proof of uniqueness • Proof by construction • Proof by cases • Proof by mathematical induction • Pigeonhole principle • Symmetry principle • Combinatorial arguments and proofs
3	Problem Solving Heuristics	Include: • Working backwards • Uncovering pattern and structure • Solving a simpler/similar problem • Considering cases • Restating the problem (e.g., contrapositive)

	Topics/Sub-topics	Content

4	Assumed Knowledge from H2 Mathematics and Additional Content	Include:
		• Functions and Graphs concepts from H2 Mathematics
		• Sequences and Series concepts from H2 Mathematics, with the following addition:
		  – Summation of series by the method of differences
		• Complex Numbers concepts from H2 Mathematics
		• Calculus concepts from H2 Mathematics, with the following addition:
		  – Reduction formulae
		  – Improper integrals
		• Probability (including counting) concepts from H2 Mathematics, with the following addition:
		  – Bijection principle (include the case of distributing indistinguishable objects into distinguishable boxes)
		  – Inclusion-Exclusion principle
		• Additional inequalities
		  – AM-GM inequality
		  – Cauchy-Schwarz inequality
		  – Triangle inequality
		• Introduction to limits
		  – Comparing polynomial, exponential, and logarithmic growth rates
		  – Operations involving limits e.g., limit of a sum is the sum of the limits, if exist
		• Concepts of congruence and modular arithmetic

5	Mathematical Investigation and Reading Mathematical Texts	Include: • Formulating a conjecture • Extension, generalisation, special cases • Complete or critique a solution

Notwithstanding the content areas defined above, candidates will also prove results and solve problems outside these defined areas or at the intersection of two or more such areas using their ability to understand and apply given definitions or results.

MATHEMATICAL NOTATION

The list which follows summarises the notation used in Singapore–Cambridge Mathematics examinations. Although primarily directed towards A-Level, the list also applies, where relevant, to examinations at all other levels.

1. *Set Notation*

$\in$	is an element of
$\notin$	is not an element of
$\{x_1, x_2, \ldots\}$	the set with elements $x_1, x_2, \ldots$
$\{x: \ldots\}$	the set of all x such that
$\mathrm{n}(A)$	the number of elements in set A
$\varnothing$	the empty set
$\mathscr{E}$	universal set
A'	the complement of the set A
$\mathbb{Z}$	the set of integers, $\{0, \pm 1, \pm 2, \pm 3, \ldots\}$
$\mathbb{Z}^+$	the set of positive integers, $\{1, 2, 3, \ldots\}$
$\mathbb{Q}$	the set of rational numbers
$\mathbb{Q}^+$	the set of positive rational numbers, $\{x \in \mathbb{Q}: x > 0\}$
$\mathbb{Q}_0^+$	the set of positive rational numbers and zero, $\{x \in \mathbb{Q}: x \geqslant 0\}$
$\mathbb{R}$	the set of real numbers
$\mathbb{R}^+$	the set of positive real numbers, $\{x \in \mathbb{R}: x > 0\}$
$\mathbb{R}_0^+$	the set of positive real numbers and zero, $\{x \in \mathbb{R}: x \geqslant 0\}$
$\mathbb{R}^n$	the real n-tuples
$\mathbb{C}$	the set of complex numbers
$\subseteq$	is a subset of
$\subset$	is a proper subset of
$\nsubseteq$	is not a subset of
$\not\subset$	is not a proper subset of
$\cup$	union
$\cap$	intersection
$[a, b]$	the closed interval $\{x \in \mathbb{R}: a \leqslant x \leqslant b\}$
$[a, b)$	the interval $\{x \in \mathbb{R}: a \leqslant x < b\}$
$(a, b]$	the interval $\{x \in \mathbb{R}: a < x \leqslant b\}$
(a, b)	the open interval $\{x \in \mathbb{R}: a < x < b\}$

2. *Miscellaneous Symbols*

$=$	is equal to
$\neq$	is not equal to
$\equiv$	is identical to or is congruent to
$\approx$	is approximately equal to
$\propto$	is proportional to
$<$	is less than
$\leqslant$; $\ngtr$	is less than or equal to; is not greater than
$>$	is greater than
$\geqslant$; $\nless$	is greater than or equal to; is not less than
∞	infinity

3. *Operations*

$\sum_{i=1}^{n} a_i$	$a_1 + a_2 + ... + a_n$
$\sqrt[n]{a}$	the *n*th root of the real number a
$\lvert a \rvert$	the modulus of the real number a
$n!$	n factorial for $n \in \mathbb{Z}^+ \cup \{0\}$, $(0! = 1)$
$\binom{n}{r}$	the binomial coefficient $\dfrac{n!}{r!(n-r)!}$, for $n, r \in \mathbb{Z}^+ \cup \{0\}$, $0 \leqslant r \leqslant n$
	$\dfrac{n(n-1)...(n-r+1)}{r!}$, for $n \in \mathbb{Q}$, $r \in \mathbb{Z}^+ \cup \{0\}$

4. *Functions*

f	the function f
$f(x)$	the value of the function f at x
$f: A \to B$	f is a function under which each element of set A has an image in set B
$f: x \mapsto y$	the function f maps the element x to the element y
f^{-1}	the inverse of the function f
gf	the composite function of f and g which is defined by $gf(x) = g(f(x))$
$\lim_{x \to a} f(x)$	the limit of $f(x)$ as x tends to a
Δx ; δx	an increment of x
$\frac{dy}{dx}$	the derivative of y with respect to x
$\frac{d^n y}{dx^n}$	the nth derivative of y with respect to x
$f'(x), f''(x), \ldots, f^{(n)}(x)$	the first, second, … nth derivatives of $f(x)$ with respect to x
$\int y \, dx$	indefinite integral of y with respect to x
$\int_a^b y \, dx$	the definite integral of y with respect to x for values of x between a and b
$\dot{x}, \ddot{x}, \ldots$	the first, second, …derivatives of x with respect to time
$\frac{\partial z}{\partial x}$	the partial derivative of z with respect to x
$\frac{\partial^2 z}{\partial y \partial x}$	the partial derivative of z with respect to x then with respect to y
f_x	the partial derivative of f with respect to x
f_{xy}	the partial derivative of f with respect to x then with respect to y

5. *Exponential and Logarithmic Functions*

e	base of natural logarithms
e^x, $\exp x$	exponential function of x
$\log_a x$	logarithm to the base a of x
$\ln x$	natural logarithm of x
$\lg x$	logarithm of x to base 10

6. *Circular Functions and Relations*

sin, cos, tan, cosec, sec, cot	the circular functions
$\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$ cosec^{-1}, $\sec^{-1}$, $\cot^{-1}$	the inverse circular functions

7. *Complex Numbers*

i	the square root of –1
z	a complex number, $z = x + \mathrm{i}y$ $= r(\cos\theta + \mathrm{i}\sin\theta),\ r \in \mathbb{R}^+$ $= r\mathrm{e}^{\mathrm{i}\theta},\ r \in \mathbb{R}^+$
Re z	the real part of z, $\mathrm{Re}\,(x + \mathrm{i}y) = x$
Im z	the imaginary part of z, $\mathrm{Im}\,(x + \mathrm{i}y) = y$
$\lvert z\rvert$	the modulus of z, $\lvert x+\mathrm{i}y\rvert = \sqrt{x^2+y^2}$, $\lvert r(\cos\theta + \mathrm{i}\sin\theta)\rvert = r$
arg z	the argument of z, $\arg(r(\cos\theta + \mathrm{i}\sin\theta)) = \theta,\ -\pi < \theta \leqslant \pi$
z^*	the complex conjugate of z, $(x + \mathrm{i}y)^* = x - \mathrm{i}y$

8. *Matrices*

$\mathbf{M}$	a matrix $\mathbf{M}$
$\mathbf{M}^{-1}$	the inverse of the square matrix $\mathbf{M}$
$\mathbf{M}^{\mathrm{T}}$	the transpose of the matrix $\mathbf{M}$
det $\mathbf{M}$	the determinant of the square matrix $\mathbf{M}$

9. *Vectors*

$\begin{pmatrix} x \\ y \end{pmatrix}$	a column vector in xy-plane
$\begin{pmatrix} x \\ y \\ z \end{pmatrix}$	a column vector in xyz-space
$\mathbf{a}$	the vector $\mathbf{a}$
$\overrightarrow{AB}$	the vector represented in magnitude and direction by the directed line segment AB
$\hat{\mathbf{a}}$	a unit vector in the direction of the vector $\mathbf{a}$
$\mathbf{i}, \mathbf{j}, \mathbf{k}$	unit vectors in the directions of the cartesian coordinate axes
$\lvert\mathbf{a}\rvert$	the magnitude of $\mathbf{a}$
$\lvert\overrightarrow{AB}\rvert$	the magnitude of $\overrightarrow{AB}$
$\mathbf{a.b}$	the scalar product of $\mathbf{a}$ and $\mathbf{b}$
$\mathbf{a}\times\mathbf{b}$	the vector product of $\mathbf{a}$ and $\mathbf{b}$

10. *Probability and Statistics*

A, B, C, etc.	events
$A \cup B$	union of events A and B
$A \cap B$	intersection of the events A and B
$\mathrm{P}(A)$	probability of the event A
A'	complement of the event A, the event 'not A'
$\mathrm{P}(A \mid B)$	probability of the event A given the event B
X, Y, R, etc.	random variables
x, y, r, etc.	value of the random variables X, Y, R, etc.
$x_1, x_2, \ldots$	observations
$f_1, f_2, \ldots$	frequencies with which the observations, $x_1, x_2, \ldots$occur
$\mathrm{p}(x)$	the value of the probability function $\mathrm{P}(X = x)$ of the discrete random variable X
$p_1, p_2, \ldots$	probabilities of the values $x_1, x_2, \ldots$of the discrete random variable X
$\mathrm{f}(x), \mathrm{g}(x)\ldots$	the value of the probability density function of the continuous random variable X
$\mathrm{F}(x), \mathrm{G}(x)\ldots$	the value of the (cumulative) distribution function $\mathrm{P}(X \leqslant x)$ of the random variable X
$\mathrm{E}(X)$	expectation of the random variable X
$\mathrm{E}[\mathrm{g}(X)]$	expectation of $\mathrm{g}(X)$
$\mathrm{Var}(X)$	variance of the random variable X
$\mathrm{B}(n, p)$	binomial distribution, parameters n and p
$\mathrm{Po}(\mu)$	Poisson distribution, mean μ
$\mathrm{Geo}(p)$	Geometric distribution, mean $\dfrac{1}{p}$
$\mathrm{N}(\mu, \sigma^2)$	normal distribution, mean μ and variance σ^2
μ	population mean
σ^2	population variance
σ	population standard deviation
$\bar{x}$	sample mean
s^2	unbiased estimate of population variance from a sample
r	linear product-moment correlation coefficient for a sample